Exponentials and Logarithms- MS

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(a) Option 1

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20000$ into their exponential model

Candidates may start by simply writing $V = 20000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares $V =$ (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20000e^{-0.223t} \Rightarrow 2000 = 20000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2

M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$

Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Uses $t = 0 \Rightarrow A = 20000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe

You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model

dM1: $t = 1 \Rightarrow 16000 = 20000r^1 \Rightarrow r = ..$ Dependent upon both previous M's

In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20000$

You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model

A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct

(b)

M1: Uses a model of the form $V = Ar^t$ oe to find the value of V when $t = 10$. Eg. 20000×0.8^{10}

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.

Compares (£)2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

(c)

B1ft: States a value of r in the range $(0.8, 1)$ or states would increase the value of "0.8"

They do not need to state that "0.8" must lie in the range $(0.8, 1)$

Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....

(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20000 \Rightarrow A = 18000$

dM1: $t = 1, V = 16000 \Rightarrow 16000 = 2000 + 18000e^k \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18000 \times e^{-0.251t} + 2000$

(b)

M1: Uses their model to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251\times10} + 2000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of k or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

Condone 'make the value of k or the -0.251 greater (or less negative)'

....

It is entirely possible that they start part (a) from a differential equation.

M1: $\frac{dV}{dt} = kV \Rightarrow \int \frac{dV}{V} = \int k dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\frac{5}{4}\right)t + \ln 20000$

2.

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to ...... \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(a)

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make 'a' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in a are on the same side of the equation and the term in b is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be corre

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \to \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$. They may state that b cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)

As a minimum accept that $b > 1$ as a cannot be negative.

Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

....

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

$$\log a - \log b = \log(a-b) \Rightarrow \log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b^2}{b-1} - b\right)$$

B1: Score for $\log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b}{b-1}\right)$

M1: Attempts to write $\frac{b^2}{b-1} - b$ as a single fraction $\frac{b^2}{b-1} - b = \frac{b^2 - b(b-1)}{b-1}$

Allow as two separate fractions with the same common denominator

A1*: Achieves lhs and rhs as $\log\left(\frac{b}{b-1}\right)$ **and** makes a comment such as "hence true"

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3.

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1	1.1b
	$A = 24795 \rightarrow 24805 \approx 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / (£) 24 800 is the value of the car on 1st January 2001	B1	3.4
	p / 1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p = \text{awrt } 1.0658$

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5, 50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \dfrac{32000}{1.0658^4}$ or $A = \dfrac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or '≈ 24800'. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.
The statement must reference **the car**, its **cost/value**, and **"0" time**
Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.
The statement must reference **a yearly rate** and **an increase in value or multiplier**.
They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."
Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'
Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their $'1.0658'^t = k$
Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\dfrac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times '1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

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4.

Question	Scheme	Marks	AOs
14 (a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{\mathrm{d}V}{\mathrm{d}t} = k\mathrm{e}^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: Achieves $\frac{\mathrm{d}V}{\mathrm{d}t} = -3925\mathrm{e}^{-0.25t}$ or $\frac{\mathrm{d}V}{\mathrm{d}t} = 15700 \times -0.25\mathrm{e}^{-0.25t}$ with both sides correct

A1*: Sets $-3925\mathrm{e}^{-0.25T} = -500$ oe and proceeds to $3925\mathrm{e}^{-0.25T} = 500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925\mathrm{e}^{-0.25T} = -500 \Rightarrow 3925\mathrm{e}^{-0.25T} = 500$ without any mention or reference to $\frac{\mathrm{d}V}{\mathrm{d}t}$

Or $15700 \times -0.25\mathrm{e}^{-0.25t} = -500 \Rightarrow 3925\mathrm{e}^{-0.25T} = 500$ without any mention or reference to $\frac{\mathrm{d}V}{\mathrm{d}t}$

(b)(ii)

M1: Proceeds from $\mathrm{e}^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$

Alternatively takes lns first $3925\mathrm{e}^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$

but $3925\mathrm{e}^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0

A1: $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24

A1: 8 years 3 months. Correct answer and solution only

Answers obtained numerically score 0 marks. The M mark must be scored.

(c)

B1: 2 300 but condone £ 2 300

(d)

B1: Any suitable reason. See scheme

Accept "Scrappage" schemes may pay more (or less) than £ 2 300.

Do not accept "does not take into account inflation"

It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

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5.

Question	Scheme		Marks	AOs
5 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$	$\frac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
				(5 marks)

(a)

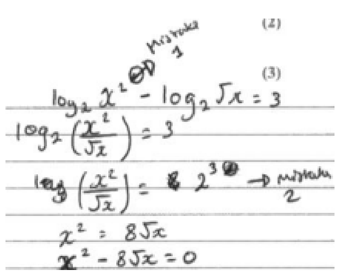
B1: States one of the two errors.

Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\frac{x^2}{\sqrt{x}}\right)$ $(=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law' Allow responses such as 'it must be $\log x^2$ before subtracting the logs'

Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**

Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0

B1: States both of the two errors. (See above)



Cases like these please send to review.

(b)

M1: Uses a correct method of combining the two log terms. Either uses both the power law and the

(b)

M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\frac{x^2}{\sqrt{x}}\right)=3$ oe. Or uses both the power law and subtraction to reach $\frac{3}{2}\log_2(x)=3$

M1: Uses correct work to "undo" the log. Eg moves from $\log_2(Ax^n)=b \Rightarrow Ax^n=2^b$

This is independent of the previous mark so allow following earlier error.

A1: cso $x=4$ achieved with at least one intermediate step shown. Extra solutions would be A0

SC: If the "answer" rather than the "solution" is given score 100.

6.

Question	Scheme	Marks	AOs
13(a)	For a correct equation in p or q $\quad p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a) This is now being marked M1 A1 M1 A1 and in this order on e pen

M1: For a correct equation in p or q This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

A1: For $p=$ awrt 63100 or $q=$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

A1: For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

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ALT I(a)

M1: Substitutes $t=0$ and states that $\log p=4.8$

A1: $p=$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p=$ awrt 63100 and $q=$ awrt 1.122

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " p **is............ " and "**q **is ...."**

(c)

M1: For substituting $t=30$ into $V = pq^t$ using their values for p and q or substituting $t=30$ into $\log_{10} V = 0.05t + 4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

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7.

Question Number	Scheme	Marks
7. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x=)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\dfrac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

	Notes
(i)	1st M1: Applies power law of logarithms correctly to one side of the equation M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a=\sqrt{16a^6}$ isw so allow $x+a=\pm 4a^3$ for Method mark. Also allow $x+a=4a^4$ or $x+a=\pm 4a^{5.5}$ or even $x+a=16a^3$ as there is evidence of attempted square root. May see the correct $x+a=10^{(\log 4+3\log a)}$ *so* $x=-a+10^{(\log 4+3\log a)}$ which gains M1A0 unless followed by the answer in the scheme. A1: Do not allow $x=\pm 4a^3-a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e.
(ii)	M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y M1: Uses $\log_3 3^2=2$ 3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y=$ A1cso: $y=\frac{10}{9}b$ or correct equivalent after **completely correct** work. **Special case:** $\frac{\log_3(9y+b)}{\log_3(2y-b)}=2$ is M0 unless clearly crossed out and replaced by the correct $\log_3\frac{(9y+b)}{(2y-b)}=2$ Candidates may then write $\frac{(9y+b)}{(2y-b)}=3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

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8.

Question Number	Scheme		Marks
8(i)	Two Ways of answering the question are given in part (i)		
Way 1	$\log_3\left(\frac{3b+1}{a-2}\right)=-1$ or $\log_3\left(\frac{a-2}{3b+1}\right)=1$	Applying the subtraction law of logarithms	M1
	$\frac{3b+1}{a-2}=3^{-1}\left\{=\frac{1}{3}\right\}$ or $\left(\frac{a-2}{3b+1}\right)=3$	Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3=a-2\Rightarrow\}\ b=\frac{1}{9}a-\frac{5}{9}$	$b=\frac{1}{9}a-\frac{5}{9}$ or $b=\frac{a-5}{9}$	A1 oe
			[3]
	In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs		
(i) **Way 2**	Either $\log_3(3b+1)-\log_3(a-2)=-\log_3 3$ or $\log_3(3b+1)+\log_3 3=\log_3(a-2)$		2nd M1
	$\log_3(3b+1)=\log_3(a-2)-\log_3 3=\log_3\left(\frac{a-2}{3}\right)$ or $\log_3 3(3b+1)=\log_3(a-2)$		1st M1
	$\{3b+1=\frac{a-2}{3}\}$ $b=\frac{1}{9}a-\frac{5}{9}$		A1
			[3]
	Five Ways of answering the question are given in part (ii)		

(ii) **Way 1** **See also common approach below in notes**	$32(2^{2x}) - 7(2^x) = 0$ — Deals with power 5 correctly giving ×32	M1
	So, $2^x = \frac{7}{32}$ — $2^x = \frac{7}{32}$ or $y = \frac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\frac{7}{32}\right)$ or $x = \frac{\log(\frac{7}{32})}{\log 2}$ or $x = \log_2\left(\frac{7}{32}\right)$ — A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
	Begins with $2^{2x+5} = 7(2^x)$ (for Way 2 and Way 3) (see notes below)	
(ii) **Way 2**	$(2x+5)\log 2 = \log 7 + x\log 2$ — Correct application of **either** the power law **or** addition law of logarithms	M1
	Correct result after applying the power **and** addition laws of logarithms.	A1
	$2x\log 2 + 5\log 2 = \log 7 + x\log 2$ $\Rightarrow x = \frac{\log 7 - 5\log 2}{\log 2}$ — Multiplies out, collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
(ii) **Way 3**	$2x + 5 = \log_2 7 + x$ — Evidence of $\log_2$ and either $2^{2x+5} \to 2x+5$ or $7(2^x) \to \log_2 7 + \log_2(2^x)$	M1
	$2x + 5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
(ii) **Way 4**	$2^{2x+5} = 7(2^x) \Rightarrow 2^{x+5} = 7$	
	$x + 5 = \log_2 7$ or $\frac{\log 7}{\log 2}$ — Evidence of $\log_2$ and either $2^{x+5} \to x+5$ or $7 \to \log_2 7$	M1
	$x + 5 = \log_2 7$ oe.	A1
	$x = \log_2 7 - 5$ — Rearranges to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
Way 5 **(similar to Way 3)**	$2^{2x+5} = 2^{\log_2 7}(2^x)$ — 7 is replaced by $2^{\log_2 7}$	M1
	$2x + 5 = \log_2 7 + x$ — $2x + 5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
		7

		Question 8 Notes
(i)	1st M1	Applying either the addition or subtraction law of logarithms correctly to combine any **two** log terms into **one** log term.
	2nd M1	For making a correct connection between log base 3 and 3 to a power.
	A1	$b=\frac{1}{9}a-\frac{5}{9}$ or $b=\frac{a-5}{9}$ o.e. e.g. Accept $b=\frac{1}{3}\left(\frac{a}{3}-\frac{5}{3}\right)$ but not $b=\frac{a-2}{9}-\frac{3}{9}$ nor $b=\frac{\left(\frac{a}{3}-\frac{5}{3}\right)}{3}$
(ii)	1st M1	First step towards solution – an equation with one side or other correct or one term dealt with correctly (see five* possible methods above)
	1st A1	Completely correct first step – giving a correct equation as shown above
	dM1	Correct complete method (all log work correct) and working to reach $x=$ in terms of logs reaching a correct expression or one where the only errors are slips solving linear equations
	2nd A1	Accept answers which round to -2.19 If a second answer is also given this becomes A0
	Special Case in (i)	Writes $\frac{\log_3(3b+1)}{\log_3(a-2)}=-1$ and proceeds to $\frac{3b+1}{a-2}=3^{-1}\left\{=\frac{1}{3}\right\}$ and to correct answer- Give M0M1A1 (special case)
	Common approach to part (ii)	Let $2^x=y$ Treat this as **Way 1** They get $32y^2-7y=0$ for M1 and need to reach $y=\frac{7}{32}$ for A1 Then back to **Way 1** as before. Any letter may be used for the new variable which I have called y. If they use x and obtain $x=\frac{7}{32}$, this may be awarded M1A0M0A0 Those who get $y^2-7y+32=0$ or $y^7-7y=0$ will be awarded M0,A0,M0,A0
	Common Present-ation of Work in ii	**Many begin with** $\log\left(2^{2x+5}\right)-\log\left(7(2^x)\right)=0$. It is possible to reach this in two stages correctly so do not penalise this and award the full marks if they continue correctly as in **Way 2**. If however the solution continues with $(2x+5)\log 2-x\log 14=0$ or with $(2x+5)\log 2-7x\log 2=0$ (both incorrect) then they are awarded M1A0M0A0 just getting credit for the $(2x+5)$ log2 term.
	Note	N.B. The answer (+)2.19 results from "algebraic errors solving linear equations" leading to $2^x=\frac{32}{7}$ and gets M1A0M1A0

9.

Question Number	Scheme	Marks
7. (i)	$8^{2x+1}=24$	
	$(2x+1)\log 8 = \log 24$ or $(2x+1)=\log_8 24$ — or $8^{2x}=3$ and so $(2x)\log 8 = \log 3$ or $(2x)=\log_8 3$	M1
	$x=\frac{1}{2}\left(\frac{\log 24}{\log 8}-1\right)$ or $x=\frac{1}{2}(\log_8 24-1)$ — $x=\frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x=\frac{1}{2}(\log_8 3)$ o.e.	dM1
	=0.264	A1 (3)
(ii)	$\log_2(11y-3)-\log_2 3-2\log_2 y=1$	
	$\log_2(11y-3)-\log_2 3-\log_2 y^2=1$	M1
	$\log_2\frac{(11y-3)}{3y^2}=1$ or $\log_2\frac{(11y-3)}{y^2}=1+\log_2 3=2.58496501$	dM1
	$\log_2\frac{(11y-3)}{3y^2}=\log_2 2$ or $\log_2\frac{(11y-3)}{y^2}=\log_2 6$ (allow awrt 6 if replaced by 6 later)	B1
	Obtains $6y^2-11y+3=0$ o.e. i.e. $6y^2=11y-3$ for example	A1
	Solves quadratic to give $y=$	ddM1
	$y=\frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1 (6) [9]

Notes (i)	**M1**: Takes logs and uses law of powers correctly. (Any log base may be used) Allow lack of brackets. **dM1**: Make x subject of their formula correctly (may evaluate the log before subtracting 1 and calculate e.g. (1.528 -1)/2) **A1**: Allow answers which round to 0.264
(ii)	**M1**: Applies power law of logarithms replacing $2\log_2 y$ by $\log_2 y^2$ **dM1**: Applies quotient or product law of logarithms correctly to the three log terms including term in y^2. (dependent on first M mark) or applies quotient rule to two terms and collects constants (allow "triple" fractions) $1+\log_2 3$ on RHS is not sufficient – need $\log_2 6$ or 2.58… e.g. $\log_2(11y-3)=\log_2 3+\log_2 y^2+\log_2 2$ becoming $\log_2(11y-3)=\log_2 6y^2$ **B1**: States or uses $\log_2 2=1$ or $2^1=2$ at any point in the answer so may be given for $\log_2(11y-3)-\log_2 3-2\log_2 y=\log_2 2$ or for $\frac{(11y-3)}{3y^2}=2$, for example (Sometimes this mark will be awarded before the second M mark, and it is possible to score M1M0B1in some cases) Or may be given for $\log_2 6=2.584962501..$ or $2^{2.584962501..}=6$ **A1**: This or equivalent quadratic equation (does not need to be in this form but should be equation) **ddM1:** (dependent on the two previous M marks) Solves their quadratic equation following reasonable log work using factorising, completion of square, formula or implied by both answers correct. **A1:** Any equivalent correct form – need both answers- allow awrt 0.333 for the answer 1/3 *NB: If "=0" is missing from the equation but candidate continues correctly and obtains correct answers then allow the penultimate A1 to be implied (Allow use of x or other varable instead of y throughout)

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10.

Question Number	Scheme	Marks
7. (i) Method 1	$\log_2\left(\dfrac{2x}{5x+4}\right) = -3$ or $\log_2\left(\dfrac{5x+4}{2x}\right) = 3$, or $\log_2\left(\dfrac{5x+4}{x}\right) = 4$ (see special case 2)	M1
	$\left(\dfrac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\dfrac{5x+4}{2x}\right) = 2^3$ or $\left(\dfrac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\dfrac{2x}{5x+4}\right)\right) = \log_2\left(\dfrac{1}{8}\right)$	M1
	$16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent)	dM1
	$x = \dfrac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	A1 **cso** **(4)**
7(i) **Method 2**	$\log_2(2x) + 3 = \log_2(5x+4)$	
	So $\log_2(2x) + \log_2(8) = \log_2(5x+4)$ (3 replaced by $\log_2 8$)	2[nd] M1
	Then $\log_2(16x) = \log_2(5x+4)$ (addition law of logs)	1[st] M1
	Then final M1 A1 as before	dM1A1
(ii)	$\log_a y + \log_a 2^3 = 5$	M1
	$\log_a 8y = 5$ Applies product law of logarithms.	dM1
	$y = \dfrac{1}{8}a^5$ $\qquad y = \dfrac{1}{8}a^5$	A1cao
		(3) **[7]**

	Notes for Question 7
(i)	1[st] M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms in** x **into one** log term in x 2[nd] M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\dfrac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. **Use of** 3^2 **is M0** 3[rd] dM1: Obtains **correct** linear equation in x. usually the one in the scheme and attempts $x =$ A1: cso Answer of 4/11 with **no** suspect log work preceding this.
(ii)	M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ *or* $\log_a 8$ dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$
(i)	**Special case 1:** $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \dfrac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \dfrac{2x}{5x+4} = 2^{-3} \Rightarrow x = \dfrac{4}{11}$ or $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \dfrac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \log_2 \dfrac{2x}{5x+4} = -3 \Rightarrow \dfrac{2x}{5x+4} = 2^{-3} \Rightarrow x = \dfrac{4}{11}$ each attempt scores M0M1M1A0 – special case
	Special case 2: $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x+4) - 3$, is M0 until the two log terms are combined to give $\log_2\left(\dfrac{5x+4}{x}\right) = 3 + \log_2 2$. This earns M1 Then $\left(\dfrac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\dfrac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before.

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11.

Question Number	Scheme		Marks
6.			
(a)	$2\log(x+15)=\log(x+15)^2$		B1
	$\log(x+15)^2-\log x=\log\dfrac{(x+15)^2}{x}$	Correct use of $\log a-\log b=\log\dfrac{a}{b}$	M1
	$2\log(x+15)-\log x=6\Rightarrow\log\left(\dfrac{(x+15)^2}{x}\right)=6$ with no incorrect work scores B1 M1 together		
	$2\log_2(x+15)-\log_2 x=2\log_2\dfrac{(x+15)}{x}$ is M0		
	$2^6=64$ or $\log_2 64=6$	64 used in the correct context	B1
	$\log_2\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=64$	Removes logs correctly	M1
	$2\log(x+15)-\log x=6\Rightarrow\log(x+15)^2-\log x=6\Rightarrow\dfrac{(x+15)^2}{x}=64$ Is acceptable for the first 4 marks		
	This method mark should only be awarded for the removal of logs in an appropriate way. Some examples are below, $\dfrac{\log(x+15)^2}{\log x}=6\Rightarrow\dfrac{(x+15)^2}{x}=6$ **M0** $\log\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=6$ **M0** $\log\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=\log_2 6$ **M0** $\log\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=6^2$ **M0** $\log\left(\dfrac{(x+15)}{x}\right)^2=6\Rightarrow\left(\dfrac{(x+15)}{x}\right)^2=64$ **M1**		
	$\Rightarrow x^2+30x+225=64x$ *or* $x+30+225x^{-1}=64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2-34x+225=0$ *	Correct completion to printed answer with no errors but allow recovery from 'invisible' brackets e.g. $x+15^2\rightarrow x^2+30x+225$	A1
			(5)

(b)	$(x-25)(x-9)=0 \Rightarrow x=25$ or $x=9$	M1: Correct attempt to solve the **given** quadratic as far as $x=$... **It must be an attempt at solving the given quadratic but allow mis-copy e.g. 255 for 225** A1: Both 25 and 9	M1 A1
			(2)
			[7]
	See appendix for some alternative correct and incorrect methods for (a)		

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12.

Question number	Scheme	Marks
2	$2\log x = \log x^2$	B1
	$\log_3 x^2 - \log_3(x-2) = \log_3 \dfrac{x^2}{x-2}$	M1
	$\dfrac{x^2}{x-2} = 9$	A1 o.e.
	Solves $x^2 - 9x + 18 = 0$ to give $x = ...$	M1
	$x = 3$, $x = 6$	A1
		Total 5
Notes	**B1** for **this** correct use of power rule (may be implied) **M1:** for **correct** use of subtraction rule (or addition rule) for logs N.B. $2\log_3 x - \log_3(x-2) = 2\log_3 \dfrac{x}{x-2}$ is **M0** **A1**. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.) **M1** for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics) **A1** for these **two** correct answers	

Alternative Method	$\log_3 x^2 = 2 + \log_3(x-2)$ is B1, *so* $x^2 = 3^{2+\log_3(x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1 Here M1 is for **complete** method i.e.correct use of powers after logs are used correctly
Common Slips	$2\log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but **the statement is M0** and so leads to no further marks $2\log_3 x - \log_3(x-2) = 2$ *so* $\log_3 x - \log_3(x-2) = 1$ and $\log_3 \frac{x}{x-2} = 1$ can earn M1 for *correct* subtraction rule following error, but no other marks
Special Case	$\frac{\log x^2}{\log(x-2)} = 2$ leading to $\frac{x^2}{x-2} = 9$ and then to x =3, x =6, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This *recovery* after uncorrected error is very common] Trial and error, Use of a table or just stating answer **with both** x=3 and x=6 should be awarded B0M0A0 then final M1A1 i.e. 2/5

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13.

Question number	Scheme	Marks
4. (a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$ $\log_3 x^2 = 2\log_3 x$ **Using** $\log_3 3 = 1$	B1 B1 B1 (3)
(b)	$3x^2 = 28x - 9$ Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1 M1 A1 (3) **6**
Notes (a)	**B1** for correct use of addition rule (or correct use of subtraction rule) **B1:** replacing $\log x^2$ by $2\log x$ – **not** $\log 3x^2$ by $2\log 3\,x$ this is **B0** **B1**. for replacing $\log 3$ by 1 (or use of $3^1 = 3$) If candidate has been awarded 3 marks and their proof includes an error or omission of reference to log*y* withhold the last mark. So just B1 B1 B0 These marks must be awarded for **work in part (a)** only	

(b)	**M1** for removing logs to get an equation in x– statement in scheme is sufficient. This needs to be accurate without any errors seen in part (b). **M1** for attempting to solve three term quadratic to give $x =$ (see notes on marking quadratics) **A1** for the two correct answers – this depends on second M mark only. Candidates often begin again in part (b) and do not use part (a). If such candidates make errors in log work in part (b) they score first **M0**. The second **M** and the **A** are earned as before. It is possible to get M0M1A1 or M0M1A0.
Alternative to (b) using y	Eliminates x to give $3y^2 - 730y + 243 = 0$ with no errors is M1 Solves quadratic to find y, then uses values to find x M1 A1 as before

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14.

Question Number	Scheme		Marks
3.	(a) $5^x = 10$ and (b) $\log_3(x-2) = -1$		
(a)	$x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$		M1
	$x\{= 1.430676558...\} = 1.43$ (3 sf)	1.43	A1 **cao**
			[2]
(b)	$(x-2) = 3^{-1}$	$(x-2) = 3^{-1}$ or $\frac{1}{3}$	M1 oe
	$x\left\{= \frac{1}{3} + 2\right\} = 2\frac{1}{3}$	$2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33	A1
			[2] **4**
(a)	M1: for $x = \dfrac{\log 10}{\log 5}$ or $x = \log_5 10$. Also allow M1 for $x = \dfrac{1}{\log 5}$ 1.43 with no working (or any working) scores M1A1 (even if left as $5^{1.43}$). Other answers which round to 1.4 with no working score M1A0. **<u>Trial & Improvement Method:</u>** M1: For a method of trial and improvement by trialing f(value between 1.4 and 1.43) = Value below 10 and f(value between 1.431 and 1.5) = Value over 10. A1 for 1.43 cao. **<u>Note</u>**: $x = \log_{10} 5$ by itself is M0; but $x = \log_{10} 5$ followed by $x = 1.430676558...$ is M1.		

(b)	M1: Is for correctly eliminating log out of the equation. **Eg 1:** $\log_3(x-2)=\log_3(\frac{1}{3}) \Rightarrow x-2=\frac{1}{3}$ only gets M1 when the logs are correctly removed. **Eg 2:** $\log_3(x-2)=-\log_3(3) \Rightarrow \log_3(x-2)+\log_3(3)=0 \Rightarrow \log_3(3(x-2))=0$ $\Rightarrow 3(x-2)=3^0$ only gets M1 when the logs are correctly removed, but $3(x-2)=0$ would score M0. **Note**: $\log_3(x-2)=-1 \Rightarrow \log_3\left(\frac{x}{2}\right)=-1 \Rightarrow \frac{x}{2}=3^{-1}$ would score M0 for incorrect use of logs. **<u>Alternative: changing base</u>** $\frac{\log_{10}(x-2)}{\log_{10}3}=-1 \Rightarrow \log_{10}(x-2)=-\log_{10}3 \Rightarrow \log_{10}(x-2)+\log_{10}3=0$ $\Rightarrow \log_{10}3(x-2)=0 \Rightarrow 3(x-2)=10^0$. At this point M1 is scored. A correct answer in (b) without any working scores M1A1.

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15.

Question Number	Scheme	Marks
7	(a) $2\log_3(x-5)=\log_3(x-5)^2$	B1
	$\log_3(x-5)^2-\log_3(2x-13)=\log_3\frac{(x-5)^2}{2x-13}$	M1
	$\log_3 3=1$ seen or used correctly	B1
	$\log_3\left(\frac{P}{Q}\right)=1 \Rightarrow P=3Q \quad \left\{\frac{(x-5)^2}{2x-13}=3 \Rightarrow (x-5)^2=3(2x-13)\right\}$	M1
	$x^2-16x+64=0$ (*)	A1 cso (5)
	(b) $(x-8)(x-8)=0 \Rightarrow x=8$ <u>Must</u> be seen in part (b). Or: Substitute $x=8$ into original equation and verify. Having additional solution(s) such as $x=-8$ loses the A mark. $x=8$ with no working scores both marks.	M1 A1 (2) 7

(a) Marks may be awarded if equivalent work is seen in part (b).

1st M: $\log_3(x-5)^2 - \log_3(2x-13) = \dfrac{\log_3(x-5)^2}{\log_3(2x-13)}$ is M0

$2\log_3(x-5) - \log_3(2x-13) = 2\log\dfrac{x-5}{2x-13}$ is M0

2nd M: After the first mistake above, this mark is available only if there is 'recovery' to the required $\log_3\left(\dfrac{P}{Q}\right) = 1 \Rightarrow P = 3Q$. Even then the final mark (cso) is lost.

'Cancelling logs', e.g. $\dfrac{\log_3(x-5)^2}{\log_3(2x-13)} = \dfrac{(x-5)^2}{2x-13}$ will also lose the 2nd M.

A typical wrong solution:

$\log_3\dfrac{(x-5)^2}{2x-13} = 1 \quad \Rightarrow \quad \log_3\dfrac{(x-5)^2}{2x-13} = 3 \quad \Rightarrow \quad \dfrac{(x-5)^2}{2x-13} = 3 \quad \Rightarrow \quad (x-5)^2 = 3(2x-13)$

(Wrong step here)

This, with no evidence elsewhere of $\log_3 3 = 1$, scores B1 M1 B0 M0 A0

However, $\log_3\dfrac{(x-5)^2}{2x-13} = 1 \Rightarrow \dfrac{(x-5)^2}{2x-13} = 3$ is correct and could lead to full marks.

(Here $\log_3 3 = 1$ is implied).

No log methods shown:

It is not acceptable to jump immediately to $\dfrac{(x-5)^2}{2x-13} = 3$. The only mark this scores is the 1st B1 (by generous implication).

(b) M1: Attempt to solve the given quadratic equation (usual rules), so the factors $(x-8)(x-8)$ with no solution is M0.

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16.

Question Number	Scheme	Marks
Q5 (a)	$\log_x 64 = 2 \quad \Rightarrow \quad 64 = x^2$	M1
	So $x = 8$	A1 (2)
(b)	$\log_2(11-6x) = \log_2(x-1)^2 + 3$	M1
	$\log_2\left[\frac{11-6x}{(x-1)^2}\right] = 3$	M1
	$\frac{11-6x}{(x-1)^2} = 2^3$	M1
	$\{11-6x = 8(x^2-2x+1)\}$ and so $0 = 8x^2 - 10x - 3$	A1
	$0 = (4x+1)(2x-3) \quad \Rightarrow x = \ldots$	dM1
	$x = \frac{3}{2}, \left[-\frac{1}{4}\right]$	A1 (6)
		[8]
(a)	M1 for getting out of logs A1 Do not need to see x = -8 appear and get rejected. Ignore x = -8 as extra solution. x= 8 with no working is M1 A1	
(b)	1st M1 for using the $n\log x$ rule 2nd M1 for using the $\log x$ - $\log y$ rule or the $\log x + \log y$ rule as appropriate 3rd M1 for using 2 to the power– need to see 2^3 or 8 (May see $3 = \log_2 8$ used) **If all three M marks have been earned and logs are still present** in equation **do not give** final M1. So solution stopping at $\log_2\left[\frac{11-6x}{(x-1)^2}\right] = \log_2 8$ would earn M1M1M0 1st A1 for a correct 3TQ 4th dependent M1 for attempt to solve or factorize their 3TQ to obtain x =… (mark depends on three previous M marks) 2nd A1 for 1.5 (ignore -0.25) s.c 1.5 only – no working – is 0 marks	
(a)	<u>Alternatives</u> Change base : (i) $\frac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or (ii) $\frac{\log_{10} 64}{\log_{10} x} = 2$, $\log x = \frac{1}{2}\log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then x = 8 is A1 **BUT** $\log x = 0.903$ so x = 8 is M1A0 (loses accuracy mark) (iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then x = 8 is A1	

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17.

Question Number	Scheme	Marks
2.(a)	$e^{3x-9} = 8 \Rightarrow 3x - 9 = \ln 8$	M1
	$\Rightarrow x = \dfrac{\ln 8 + 9}{3}, = \ln 2 + 3$	A1, A1
		(3)
(b)	$\ln(2y+5) = 2 + \ln(4-y)$	
	$\ln\left(\dfrac{2y+5}{4-y}\right) = 2$	M1
	$\left(\dfrac{2y+5}{4-y}\right) = e^2$	M1
	$2y + 5 = e^2(4-y) \Rightarrow 2y + e^2 y = 4e^2 - 5 \Rightarrow y = \dfrac{4e^2 - 5}{2 + e^2}$	dM1, A1
		(4)
		7 marks

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8 + 9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3 + \ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3 \log e} + 3$ or even 3.69

A1 cso $\ln 2 + 3$. Accept $\ln 2e^3$

Alt I (a)

$e^{3x-9} = 8 \Rightarrow \dfrac{e^{3x}}{e^9} = 8 \Rightarrow e^{3x} = 8e^9 \Rightarrow 3x = \ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3} = \sqrt[3]{8} \Rightarrow x - 3 = \ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9} = 2 \Rightarrow x - 9 = \ln 2$)

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2 + \ln(4-y) = \ln\left(e^2(4-y)\right)$ or $\ln(2y+5) - \ln(4-y) = \ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the e^2

M1 Scored for an attempt to undo the ln's to get an equation in y This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y)) = 2 \Rightarrow g(y) = e^2$ and can be scored eg where $g(y) = 2y + 5 - (4-y)$

It cannot be awarded for just $2y + 5 = e^2 + 4 - y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making y the subject. Expect to see both terms in y collected and factorised (may be implied) before reaching y =. Condone slips, for eg, on signs. y =2.615 scores this.

A1 $y = \dfrac{4e^2 - 5}{2 + e^2}$ or equivalent such as $y = 4 - \dfrac{13}{2 + e^2}$ ISW after you see the correct answer.

Special Case: $\ln(2y+5)-\ln(4-y)=2 \Rightarrow \dfrac{\ln(2y+5)}{\ln(4-y)}=2 \Rightarrow \dfrac{2y+5}{4-y}=e^2 \Rightarrow$ Correct answer score M0 M1 M1 A0

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18.

Question Number	Scheme	Marks
2.(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$	M1
	$\Rightarrow x=\dfrac{e^5-1}{2}$	A1
		(2)
(b)	$3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$	
	$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$ oe	A1
		(4) **6 marks**
Alt 1 2(b)	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$	A1
		(4)

Alt 2 2(b) Using logs	$3^x e^{4x} = e^7 \Rightarrow \log(3^x e^{4x}) = \log e^7$ $\log 3^x + \log e^{4x} = \log e^7 \Rightarrow x\log 3 + 4x\log e = 7\log e$ $x(\log 3 + 4\log e) = 7\log e \Rightarrow x = ...$ $x = \dfrac{7\log e}{(\log 3 + 4\log e)}$	 M1, M1 dM1 A1 (4)
Alt 3 2(b) Using $\log_3$	$3^x e^{4x} = e^7 \Rightarrow 3^x = \dfrac{e^7}{e^{4x}}$ $3^x = e^{7-4x} \Rightarrow x = (7-4x)\log_3 e$ $x(1 + 4\log_3 e) = 7\log_3 e \Rightarrow x = ...$ $x = \dfrac{7\log_3 e}{(1 + 4\log_3 e)}$	 M1,M1 dM1 A1 (4)
Alt 4 2(b) Using $3^x = e^{x\ln 3}$	$3^x e^{4x} = e^7 \Rightarrow e^{x\ln 3} e^{4x} = e^7$ $\Rightarrow e^{x\ln 3 + 4x} = e^7, \Rightarrow x\ln 3 + 4x = 7$ $x(\ln 3 + 4) = 7 \Rightarrow x = ...$ $\quad x = \dfrac{7}{(\ln 3 + 4)}$	 M1,M1 dM1 A1 (4)

(a)

M1 Proceeds from $2\ln(2x+1) - 10 = 0$ to $\ln(2x+1) = 5$ before taking exp's to achieve x in terms of e^5

Accept for M1 $2\ln(2x+1) - 10 = 0 \Rightarrow \ln(2x+1) = 5 \Rightarrow x = f(e^5)$

Alternatively they could use the power law before taking exp's to achieve x in terms of $\sqrt{e^{10}}$

$2\ln(2x+1) = 10 \Rightarrow \ln(2x+1)^2 = 10 \Rightarrow (2x+1)^2 = e^{10} \Rightarrow x = g\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x = \dfrac{e^5 - 1}{2}$ or other exact simplified alternatives such as $x = \dfrac{e^5}{2} - \dfrac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\dfrac{\sqrt{e^{10}} - 1}{2}$ does not score this mark unless simplified. $x = \dfrac{\pm e^5 - 1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x}) = \ln 3^x + \ln e^{4x}$ or $\ln(3^x e^{4x}) = \ln 3^x + 4x$ is evidence for the addition law

If the e^{4x} was 'moved' over to the right hand side score for either e^{7-4x} or the subtraction law.

$\ln\frac{e^7}{e^{4x}} = \ln e^7 - \ln e^{4x}$ or $3^x e^{4x} = e^7 \Rightarrow 3^x = \frac{e^7}{e^{4x}} \Rightarrow 3^x = e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or e^{7-4x}).

$\ln 3^x + \ln e^{4x} = \ln e^7 \Rightarrow x\ln 3 + 4x\ln e = 7\ln e$ is an example after the addition law

$3^x = e^{7-4x} \Rightarrow x\log 3 = (7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x} = e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x}) = \ln e^7 \Rightarrow x\ln 3\times 4x\ln e = 7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in x and proceeds to $x =$.
Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e = 1$ and achieve $x = \frac{7}{(\ln 3 + 4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x = \frac{7\log e}{(\log 3 + 4\log e)}$ or other exact equivalents such as $x = \frac{7\log e}{\log 3e^4}$.

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19.

Question Number	Scheme	Marks
6(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1, M1
	So $36 - 30x + 6x^2 = x^2 + 2x + 1$ and $5x^2 - 32x + 35 = 0$	A1
	Solve $5x^2 - 32x + 35 = 0$ to give $x = \frac{7}{5}$ oe (Ignore the solution $x = 5$)	M1A1
		(5)
(b)	Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x\ln 2 + (3x+1)\ln e = \ln 10$	M1
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses $\ln e = 1$	M1
	$x = \frac{-1 + \ln 10}{3 + \ln 2}$	A1
		(5)
	Note that the 4th M mark may occur on line 2	**(10 marks)**

Notes for Question 6

(a)

M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2\ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method.
The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.

A1 Deduces x = 1.4 or equivalent. Accept both x=1.4 and x=5. Candidates do not have to eliminate $x=5$. You may ignore any other solution as long as it is not in the range $-1<x<2$. Extra solutions in the range scores A0.

Notes for Question 6 Continued

(b)

M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law.

M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1

$$\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x\ln 2 \times (3x+1)\ln e = \ln 10$$

dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in x. The terms in x must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to x = …

M1 Uses ln e = 1. This could appear in line 2, but it must be part of their equation and not just a statement.

Another example where it could be awarded is $e^{3x+1}=\frac{10}{2^x}\Rightarrow 3x+1=...$

A1 Obtains answer $x=\frac{-1+\ln 10}{3+\ln 2}=\left(\frac{\ln 10-1}{3+\ln 2}\right)=\left(\frac{\log_e 10-1}{3+\log_e 2}\right)oe$. **DO NOT ISW HERE**

Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5.

$$\text{Answer} = x=\frac{-\log e+\log 10}{3\log e+\log 2}=\left(\frac{-\log e+1}{3\log e+\log 2}\right)$$

Note 2: If the candidate writes $x = \dfrac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

Question Number	Scheme	Marks
Alt 1 to 6(b)	Writes lhs in e's $2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2} e^{3x+1} = 10$	1st M1
	$\Rightarrow e^{x\ln 2+3x+1} = 10, \quad x\ln 2+3x+1 = \ln 10$	2nd M1, 4th M1
	$x(\ln 2+3) = \ln 10 - 1 \Rightarrow x = ..$	dM1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1 **(5)**

Notes for Question 6 Alt 1

M1 Writes the lhs of the expression in e's. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln's of both sides to produce and attempt to solve a linear equation in x
You may condone slips in signs for this mark but the process must be correct leading to x= ..

M1 Uses ln e = 1. This could appear in line 2

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20.

Question Number	Scheme	Marks
8. (a)	$\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$	M1 B1 A1 **aef** (3)
(b)	$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right) = 1$	M1
	$\dfrac{2x^2+9x-5}{x^2+2x-15} = e$	dM1
	$\dfrac{2x-1}{x-3} = e \Rightarrow \quad 3e-1 = x(e-2)$	M1
	$\Rightarrow x = \dfrac{3e-1}{e-2}$	A1 **aef cso**
		(4) **[7]**

	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right)=1.$ The product law of logarithms can be used to achieve $\ln\left(2x^2+9x-5\right)=\ln\left(e\left(x^2+2x-15\right)\right).$ The product and quotient law could also be used to achieve $\ln\left(\frac{2x^2+9x-5}{e(x^2+2x-15)}\right)=0.$ dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\frac{3e-1}{e-2}$ or $\frac{3e^1-1}{e^1-2}$ or $\frac{1-3e}{2-e}$. **aef** Note that the answer needs to be in terms of e. The decimal answer is 9.9610559… Note that the solution must be correct in order for you to award this final accuracy mark. **Note: See Appendix for an alternative method of long division.**	

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21.

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x-7)=5$		
	$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1
		Then rearranges to make x the subject.	dM1
	$3x-7=e^5 \Rightarrow x=\frac{e^5+7}{3}$ $\{=51.804...\}$	***Exact answer*** of $\frac{e^5+7}{3}$.	A1
			(3)

(b)	$3^x e^{7x+2} = 15$		
	$\ln\left(3^x e^{7x+2}\right) = \ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x + \ln e^{7x+2} = \ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3 + 7x + 2 = \ln 15$	$x\ln 3 + 7x + 2 = \ln 15$	A1 oe
	$x(\ln 3 + 7) = -2 + \ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x = \dfrac{-2 + \ln 15}{7 + \ln 3}$ $\{= 0.0874...\}$	***Exact answer*** of $\dfrac{-2 + \ln 15}{7 + \ln 3}$	A1 oe
			(5)
(ii) (a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$		
	$y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$ $\Rightarrow \ln(y-3) = 2x$ $\Rightarrow \frac{1}{2}\ln(y-3) = x$	Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides	M1 M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y) = \frac{1}{2}\ln(y-3)}$ (see appendix)	A1 cao
	$f^{-1}(x)$: Domain: $\underline{x > 3}$ or $\underline{(3, \infty)}$	Either $\underline{x > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Domain} > 3}$.	B1
			(4)
(b)	$g(x) = \ln(x-1), x \in \mathbb{R}, x > 1$		
	$fg(x) = e^{2\ln(x-1)} + 3$ $\{= (x-1)^2 + 3\}$	An attempt to put function g into function f. $e^{2\ln(x-1)} + 3$ or $(x-1)^2 + 3$ or $x^2 - 2x + 4$.	M1 A1 isw
	$fg(x)$: Range: $\underline{y > 3}$ or $\underline{(3, \infty)}$	Either $\underline{y > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Range} > 3}$ or $\underline{fg(x) > 3}$.	B1
			(3)
			[15]